Design of Secure, Spatially-Distributed, Data-Driven Control and Optimization Algorithms

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# Table of Contents

## 1. Introduction to Cooperative Control
- Background
- Standard Results
- Consensus Laws

## 2. Nonlinear Plug-and-Play Control Design
- Passive and Passivity-Short (PS) Systems
- Plug & Play Networked Operation of PS Systems
- Application to Attitude Synchronization
- Control of Interconnected Heterogeneous Systems Using an Information Graph

## 3. Resilient Networked Control Against Attacks
- A Dynamic Attack Model
- Robustification: Virtual Nodes and Synthetic Anchors
- Robustification of Cooperative Systems Against Attacks
**Networked Operation of Heterogeneous Dynamic Systems**

Why "distributed intelligence"?
- Information is distributed
- Actions are distributed
  - more efficient
  - more resilient
- Plug-and-play operation is essential for many operations

Consensus: all the "chosen" state variables (i.e., outputs) converge to the same value (or certain property). Applications:
- synchronization (velocity, frequency, etc)
- rendezvous, formation control, ...
- fair distribution of resources, ...
- distributed estimation of certain quantity
- distributed search for a global optimal solution of any kind

Consensus can be reached by collaborative entities (by employing cooperative control or distributed optimization or distributed estimation)

Applicable to multi-entity cooperative games
Cooperative Behaviors: Consensus

Bird flocking:

Formation flying of UAVs:
Table of Contents

1 Introduction to Cooperative Control
   - Background
   - Standard Results
   - Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3 Resilient Networked Control Against Attacks
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
Standard Stability Analysis

Linear systems: given $x(t_0) = x_0$,

$$
\dot{x} = Ax, \quad x(t_0) = x_0;
$$
or

$$
x_{k+1} = Dx_k.
$$

Eigen-analysis: solutions $\lambda_i$ from

$$
det[\lambda I - A] = 0, \quad det[\lambda I - D] = 0.
$$

Stability: system is asymptotically stable if, for all $i$,

$$
Re[\lambda_i(A)] < 0, \quad |\lambda_i(D)| < 1.
$$

Example:

$$
\dot{x} = Ax, \quad A = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix}.
$$

$$
\Delta(\lambda I - A) = \lambda^2 + k_v \lambda + k_p = 0.
$$

The system is asymptotically stable if and only if $k_v, k_p > 0$. 
Example: Simple Systems

Spring-damper system:

\[ m\ddot{x} = -b\dot{x} - kx + F, \]

where \( F \) is the control, \( b \) is the damping, and \( k \) is the spring constant.

Typical tracking control is:

\[ F = \ddot{x}^d + b\dot{x}^d + kx^d. \]

The closed loop system becomes: \( e = x - x^d \),

\[ \ddot{e} + k_v \dot{e} + k_p e = 0, \]

where \( k_v = d/m \) and \( k_p = k/m \).

Question: What happens if the systems are interconnected intermittently?
Stability and Robustness under Switching

Question: What systems can be networked in a plug-and-play manner?
# Table of Contents

1. Introduction to Cooperative Control
   - Background
   - Standard Results
   - Consensus Laws

2. Nonlinear Plug-and-Play Control Design
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3. Resilient Networked Control Against Attacks
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
A Simple Cooperative System

First-order (agent) model: \( x \in \mathbb{R}^n \),

\[
\dot{x}_i = u_i, \quad \implies \quad \dot{x} = u.
\]

Communication network model:

\[
S(t) = [s_{ij}(t)] \in \mathbb{R}_+^{n \times n}, \quad s_{ii} \equiv 1.
\]

Leaderless linear cooperative control design (consensus law):

\[
u_i = \sum_{j=1}^{n} s_{ij}(t)[x_j(t) - x_i(t)], \quad \text{or} \quad u = -L(t)x(t),
\]

where \( L \in \mathbb{R}^{n \times n} \) is the Laplacian:

\[
L_{ij}(t) = \begin{cases} 
-s_{ij}(t) & i \neq j \\
\sum_{l \neq i} s_{il}(t) & i = j 
\end{cases}
\]

Conclusions:
(a) No matter what \( S(t) \), the system is uniformly bounded.
(b) If the cumulative graph of \( S(t) \) is strongly connected, consensus is reached.
Why Stability/Consensus Is Guaranteed?

Closed-loop dynamics of the $i$th agent:

$$\dot{x}_i = \sum_{j=1}^{n} s_{ij}(t) [x_j(t) - x_i(t)].$$

Let $i^*$ be the index such that

$$x_{i^*}(t) = \max_i x_i(t),$$

then, for all $j$,

$$s_{i^*j}(t) [x_j(t) - x_{i^*}(t)] \leq 0$$

and the inequality is strict unless $s_{i^*j}(t) = 0$ or $x_j(t) = x_{i^*}(t)$. Therefore,

$$\dot{x}_{i^*} \leq 0,$$

and the maximum never increases and it always decreases if there is any connection with non-maximum agents.

Similarly, the minimum never decreases and it always increases if there is any connection with non-minimum agents.
A Leader-Followers Design

First-order (agent) model: $x \in \mathbb{R}^n,$

$$\dot{x}_i = u_i, \quad \Rightarrow \quad \dot{x} = u.$$ 

The leader state is designated as $x_0$, and communication matrices are

$$S(t) = [s_{ij}(t)] \in \mathbb{R}_+^{n \times n}, \quad s_{ii} \equiv 1, \quad s_{i0}(t) \in \mathbb{R}_+, \quad s_{0i} \equiv 0.$$ 

Leader-follower linear cooperative control design (consensus law):

$$u_i = s_{i0}[x_0 - x_i(t)] + \sum_{j=1}^{n} s_{ij}(t)[x_j(t) - x_i(t)].$$

Conclusions:
(a) No matter what $s_{ij}(t)$, the system is uniformly bounded.
(b) If the cumulative graph is strongly connected, consensus of $x_0$ is reached.

How about more general classes of systems?
Table of Contents

1 Introduction to Cooperative Control
   - Background
   - Standard Results
   - Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3 Resilient Networked Control Against Attacks
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
Dissipativity Theory

Heterogenous systems:

\[
\dot{z}_i = F_i(z_i, v_i), \quad y_i = H_i(z_i),
\]

where

\[
v_i = v_{s_i}(z_i) + u_i.
\]

Dissipativity (Willems): for p.s.d. storage function \(V_i\) and a supply rate \(\Phi_i(\cdot)\),

\[
V_i(z_i) - V_i(z_i(0)) \leq -\int_0^t \Phi_i(z_i, u_i) ds,
\]

where, if \(\Phi_i(\cdot)\) is quadratic,

\[
\Phi_i(z_i, u_i) = -\eta_i(z_i) + u_i^T y_i + \frac{\epsilon_i}{2} \|u_i\|^2 - \frac{\rho_i}{2} \|y_i\|^2.
\]

Common forms of dissipativity:

- **Passivity**: \(\eta_i(\cdot)\) p.s.d., \(\epsilon_i \leq 0\) and \(\rho_i \geq 0\).
- **\(L_2\) gain**: \(\eta_i(\cdot)\) p.s.d., \(\rho_i > 0\), and \(\epsilon_i > 0\).
- **Passivity shortage (PS)**:
  Input PS (input-feedforward passive): \(\eta_i(\cdot)\) p.s.d., \(\epsilon_i > 0\), and \(\rho_i \geq 0\).
  Output PS (output-feedback passive): \(\eta_i(\cdot)\) p.s.d., \(\epsilon_i \leq 0\), and \(\rho_i < 0\).
  PS: \(\eta_i(\cdot)\) p.s.d., \(-1 \leq \epsilon_i \rho_i < 0\).
Passive Systems

Heterogenous systems:
\[ \dot{z}_i = \mathcal{F}_i(z_i, v_i), \quad y_i = H_i(z_i), \]
where \( z_i \in \mathbb{R}^{n_i}, v_i, y_i \in \mathbb{R}^m, \partial H(z_i) / \partial z_i \) has rank \( m \), and
\[ v_i = v_{si}(z_i) + u_i. \]

Passivity: for p.s.d. storage function \( V_i \) and p.s.d. function \( \eta_i \),
\[ V_i(z_i) - V_i(z_i(0)) \leq -\int_0^t \eta_i(z_i) ds + \int_0^t u_i^T y_i ds. \]

Linear systems:
\[ \dot{z}_i = F_{i}^{c}z_i + G_i u_i, \quad y_i = H_i z_i, \]
- positive real
- relative-degree-one
- minimum-phase
- Lyapunov function \( P_i: G_i^T P_i = H_i \)
Heterogenous systems:

\[ \dot{z}_i = \mathcal{F}_i(z_i, v_{s_i}(z_i)) + u_i, \quad y_i = H_i(z_i). \]

Input passivity-short system: for p.s.d. storage function \( V_i \), p.s.d. function \( \eta_i \) and constant \( \epsilon_i \geq 0 \),

\[ V_i(z_i) - V_i(z_i(0)) \leq -\int_0^t \eta_i(z_i) ds + \int_0^t u_i^T y_i ds + \frac{\epsilon_i}{2} \int_0^t \|u_i\|^2 ds, \]

where \( \epsilon_i \) is the so-called impact coefficient.

\(-\epsilon_i/2\) is the slope above; as \( \epsilon_i \to \infty \), the whole plane is covered.

(a) Most Lyapunov stable systems
(b) All Lyapunov-stable linear systems can be made PS under an output feedback control
Heterogenous systems:

\[ \dot{z}_i = \mathcal{F}_i(z_i, v_{s_i}(z_i) + u_i), \quad y_i = H_i(z_i). \]

The \( i \)th system is said to be PS and have an \( L_2 \)-gain \( \varrho_i \) if, for some \( \varrho_i, \epsilon_i > 0 \),

\[ V_i(z_i) - V_i(z_i(0)) \leq -\frac{\varrho_i}{2} \int_0^t \| y_i \|^2 ds + \int_0^t u_i^T y_i ds + \frac{\epsilon_i}{2} \int_0^t \| u_i \|^2 ds. \]

For linear system \( G(s) \), it follows that

\[ \varrho_i \left[ \left( \text{Re}[G(j\omega)] - \frac{1}{\varrho_i} \right)^2 + \text{Im}^2[G(j\omega)] - \frac{1}{\varrho_i^2} \right] \leq \epsilon_i. \]

If \( \varrho_i \to 0 \), we recover the input PS result.

Admissible area: Inside the cycle centered at \( 1/\varrho_i \) and of radius \( \sqrt{\epsilon_i/\varrho_i + 1/\varrho_i^2} \).

For a large enough value of \( \epsilon \), the circle will contain Nyquist plot of any Hurwitz system.
Physical Meanings of $\epsilon$ and $\varrho$

Consider the linear system

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad A = \begin{bmatrix} 0 & 1 \\ -w_n^2 & -2\xi w_n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T.$$ 

It follows from Lyapunov function

$$V = x^T P x, \quad P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix},$$

that

$$\dot{V} \leq -\frac{\varrho}{2} y^2 + uy + \frac{\epsilon}{2} u^2,$$

if and only if $p_1 > 0, p_1 p_3 > p_2^2$ and

$$W \triangleq \begin{bmatrix} -2w_n^2 p_2 + 0.5\varrho & p_1 - 2\xi w_n p_2 - w_n^2 p_3 & p_2 - 0.5 \\ p_1 - 2\xi w_n p_2 - w_n^2 p_3 & 2(p_2 - 2\xi w_n p_3) & p_3 \\ p_2 - 0.5 & p_3 & -0.5\epsilon \end{bmatrix} < 0.$$ 

Solutions of $\epsilon$ and $\varrho$ using Lyapunov function from $A^T P + PA = \text{diag}\{-w_n, -1/w_n\} I$:

$$\varrho = w_n \quad \Rightarrow \quad \epsilon = \frac{1}{w_n} \left(1 - \frac{1}{w_n}\right)^2 + 2 \frac{1}{(2\xi w_n)^2 w_n}.$$ 

As $\xi$ becomes smaller, $\epsilon$ increases.
Why PS Systems but not Passive Systems?

Some systems are passive:
- First-order Hurwitz systems (e.g., $1/(s + a)$) are passive.
- Parallel connection of two passive systems is passive.
- Negative loop connection of two passive systems is passive.

Most high-order systems are not passive even though they consist of passive elements:
- Serial connection of two passive systems is generally not passive, e.g., $1/(s + 1)^2$.
- Time delay is not passive.
- Discretization of passive systems is typically not passive, e.g., $1/s$.

Passive short (PS) systems form a much broader of systems:
- All Lyapunov stable linear systems are either PS or can be made PS under an output feedback control.
- PS systems can be interconnected: parallel, series, negative feedback loop, positive feedback loop, etc.

The passive short framework applies directly to discretized systems.
Consider a positive feedback connection of two PS systems. Cooperative stability (consensus) is ensured if

\[ 0 < k_y < \frac{2}{(\epsilon_i + \epsilon_j)}. \]

Impact on networked operation is revealed. Asymptotic stability could be further concluded if the systems are zero-state observable.

Proof: Consider

\[ V \triangleq V_i + V_j, \quad u_i = k_y(y_j - y_i), \quad \text{and} \quad u_j = k_y(y_i - y_j). \]

Then,

\[
V \leq \sum_{k=i,j} \left[ V_k(z_k(0)) - \int_0^t \left[ \eta_k(z_k) + u_k^T y_k + \frac{\epsilon_k}{2} \| u_k \|^2 \right] ds \right]
\]

\[
\leq V(0) - \frac{1}{2} \left[ 2 - (\epsilon_i + \epsilon_j)k_y \right] k_y \int_0^t \| y_i - y_j \|^2 ds.
\]

Hence, we have

\[ z_i, z_j \in L_\infty, \quad (y_i - y_j) \in L_2, \quad (y_i - y_j) \to 0. \]

Other interconnections (e.g., negative feedback connection) work as well.
Table of Contents

1 Introduction to Cooperative Control
   • Background
   • Standard Results
   • Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   • Passive and Passivity-Short (PS) Systems
   • Plug & Play Networked Operation of PS Systems
   • Application to Attitude Synchronization
   • Control of Interconnected Heterogeneous Systems Using an
     Information Graph

3 Resilient Networked Control Against Attacks
   • A Dynamic Attack Model
   • Robustification: Virtual Nodes and Synthetic Anchors
   • Robustification of Cooperative Systems Against Attacks
Control of Uncoupled Heterogeneous Systems Using an Information Graph

Communication graph: $S(t)$ with $S_{ii} \equiv 1$ and $S_{ij} \in \{0, 1\}$.

Network topology: Laplacian $L(t)$, where $D = \text{diag}\{S1\}$ and $L = (D - S)$.

Heterogeneous systems:

$$\dot{z}_i = F_i(z_i, v_i), \quad y_i = H_i(z_i),$$

each of which is passivity short with impact coefficient $\epsilon_i$.

Cooperative control protocol:

$$u_i = k_{yi} \sum_j (y_j - y_i)S_{ij}.$$

Impact Equivalence Principle: As for fictitious systems $\dot{y}_i = u_i$, consensus is ensured if the graph has at least one global reachable node and $k_{yi} \leq \bar{k}_y$ (which depends upon $\max \epsilon_i$).

If $L$ is irreducible and fixed,

$$\bar{k}_y = \frac{\lambda_2(\Gamma L + L^T \Gamma)}{2(\max \epsilon_i)\lambda_{max}(L^T \Gamma L)}.$$

where $\gamma_1^T L = 0$, $\Gamma$ is a diagonal matrix of entries in $\gamma_1$.

Advantages: networked systems, modularized design, plug-and-play operation.
Table of Contents

1 Introduction to Cooperative Control
   • Background
   • Standard Results
   • Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   • Passive and Passivity-Short (PS) Systems
   • Plug & Play Networked Operation of PS Systems
   • Application to Attitude Synchronization
   • Control of Interconnected Heterogeneous Systems Using an Information Graph

3 Resilient Networked Control Against Attacks
   • A Dynamic Attack Model
   • Robustification: Virtual Nodes and Synthetic Anchors
   • Robustification of Cooperative Systems Against Attacks
Example: Discrete-time 3-D Attitude Synchronization

Special Orthogonal group: \( SO(3) \triangleq \{ R \in \mathbb{R}^{3 \times 3} \mid RR^T = I_3, \ \det(R) = 1 \} \).

Given inertial frame \( \sum_w \), attitude of body \( i \) (body frame \( \sum_i \)) is denoted by exponential coordinate \( e^{\hat{\xi}_w \theta_w} \), where \( \xi_w \in \mathbb{R}^3 \) and \( \theta_w \in (-\pi + \epsilon, \pi - \epsilon) \) are the axis and angle of the rotation matrix, respectively.

\[ so(3) \triangleq \{ S \in \mathbb{R}^{3 \times 3} \mid S^T = -S \} \] is the Lie algebra of \( SO(3) \).

Operator \( \hat{\cdot} : \mathbb{R}^3 \to so(3) \) (whose inverse is denoted by \( \hat{\cdot} \)) is defined by

\[
\hat{a} = (a)^\triangleq \begin{bmatrix}
0 & -a_3 & a_2 \\
a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix}.
\]
Angular velocity of body $i$ with respect to $\sum_w$ is

$$
\omega^b_{wi} \triangleq \left( e^{-\hat{\xi}_\theta wi} \hat{\xi}_\theta wi \right)^\sim.
$$

A precise discrete-time model of rigid body motion to preserve the $SO(3)$ structure is

$$
e^{\hat{\xi}_\theta wi}(k+1) = e^{\hat{\xi}_\theta wi}(k) e^{h\hat{\omega}^b_{wi}(k)}.
$$

Passivity-short property:

$$
\phi(e^{\hat{\xi}_{ij}(k+1)}) - \phi(e^{\hat{\xi}_{ij}(k)}) \leq \left( sk(e^{h\hat{\omega}^b_{wi}(k)}^\sim) \right)^T \left( sk(e^{\hat{\xi}_\theta wi}(k))^\sim \right) + \| sk(e^{h\hat{\omega}^b_{wi}(k)}^\sim) \|^2,
$$

where

$$
\begin{align*}
sk(e^{\hat{\xi}_\theta}) &= \frac{1}{2}(e^{\hat{\xi}_\theta} - e^{-\hat{\xi}_\theta}), & sk(e^{\hat{\xi}_\theta})^\sim &= \hat{\xi}\sin \theta, & sk(e^{h\hat{\omega}^b_{wi}(k)}^\sim) &= \omega^b_{wi} \frac{\sin(h\|\omega^b_{wi}\|)}{\|\omega^b_{wi}\|}.
\end{align*}
$$
Discrete-time 3-D Attitude Synchronization Law

Control objective:

\[ \lim_{k \to \infty} \phi(e^{\hat{\xi}_{ij}(k)}) = 0, \]

where

\[ e^{\hat{\xi}_{ij}} \triangleq e^{-\hat{\xi}_{wi} e^{\hat{\xi}_{wj}}}, \quad \phi(e^{\hat{\xi}_{ij}}) \triangleq \frac{1}{4} \| I_3 - e^{\hat{\xi}_{ij}} \|_F^2 = \frac{1}{2} \text{tr}(I_3 - e^{\hat{\xi}_{ij}}). \]

Discrete-time synchronization law (based on passivity shortage):

\[ \sk(e^{h\hat{\omega}_{wi}^b(k)}) = h k_i \sum_{j \in \mathcal{N}_i} w_{ij} \sk(e^{h\hat{\xi}_{ij}(k)}). \]

Stability condition:

\[ k_i < \frac{\sin \epsilon}{2 h |\mathcal{N}_i| \max_{j \in \mathcal{N}_i} w_{ij}}. \]
Simulation of Discrete-time 3-D Attitude Synchronization

20 rigid bodies with $h = 0.02$ sec, randomly generated initial conditions, $\epsilon = \pi/10$: 

\begin{align*}
\theta_{wi,x} & \quad \text{[rad]} \\
\theta_{wi,y} & \quad \text{[rad]} \\
\theta_{wi,z} & \quad \text{[rad]}
\end{align*}
1. Introduction to Cooperative Control
   - Background
   - Standard Results
   - Consensus Laws

2. Nonlinear Plug-and-Play Control Design
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3. Resilient Networked Control Against Attacks
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
Interconnected Heterogeneous Systems

Interconnected heterogeneous systems:

\[ \dot{x}_i = A_i(x_i)x_i + B_i(x_i)v_i + \sum_{j \in \mathcal{N}_i} H_{ij}(y_i, y_j)(y_j - y_i), \quad y_i = C_i(x_i)x_i. \]

Local control: \( v_i = -K_i(x_i)x_i + u_i \), which yields \( \bar{A}_i(x_i) = A_i(x_i) - B_i(x_i)K_i(x_i) \).

If

\[
\begin{bmatrix}
\bar{A}_i^T(x_i)P_i + P_i\bar{A}_i(x_i) + \rho_iC_i^T C_i \\
- \sum_{j \in \mathcal{N}_i} (P_i H_{ij} C_i + C_i^T H_{ij}^T P_i) \\
\vdots \\
H_{ij}^T(y_i, y_j)P_i \\
\vdots \\
B_i^T P_i - C_i
\end{bmatrix} \leq 0,
\]

then

\[
\dot{V}_i \leq u_i^T y_i + \frac{\epsilon_{ii}}{2} \|u_i\|^2 - \frac{\rho_i}{2} \|y_i\|^2 + \frac{1}{2} \sum_{j \in \mathcal{N}_i} \epsilon_{ij} \|y_j\|^2.
\]
Modular and Data-Driven Control Design

Network-level model:

\[ \dot{V}_i \leq u_i^T y_i + \varepsilon_{ii} \frac{1}{2} \| u_i \|^2 - \rho_i \frac{1}{2} \| y_i \|^2 + \frac{1}{2} \sum_{j \in \mathcal{N}_i} \varepsilon_{ij} \| y_j \|^2. \]

Local design criterion:

\[ \min \sum_{j \in \mathcal{N}_i} \alpha_{ij} \varepsilon_{ij} \]
subject to \( \alpha_{ij} > 0 \) and \( \overline{M}_i(x_i, y_j) \leq 0 \).

Networked control on an information graph:

\[ u_i = -k_{ci} \sum_j S_{ij}^c(t)(y_j - y_i) = -K_c L y. \]

Cooperative control design criterion: Find \( k_{ci} \) such that

\[ Q = \Gamma L^T + L \Gamma - L^T K_c W L + \Psi > 0, \]

where \( \Psi = \text{diag}\{\psi_i\} \) with

\[ \psi_i = \frac{\gamma_i \rho_i}{k_{ci}} - \sum_{j=1:n; i \in \mathcal{N}_j} \frac{\gamma_j}{k_{cj}} \varepsilon_{ji}, \]

Advantages: networked systems, modularized design, plug-and-play operation, and data-driven.
# Table of Contents

1. **Introduction to Cooperative Control**
   - Background
   - Standard Results
   - Consensus Laws

2. **Nonlinear Plug-and-Play Control Design**
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3. **Resilient Networked Control Against Attacks**
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
Destabilizing Attacks

A system under attack:

\[
\begin{align*}
\dot{y} &= -L_s y + L_a d, \\
\dot{d} &= F_a d + B_a y
\end{align*}
\]

Consider the case \( L_a = L_s \) and the injection model is a low-pass filter with \( F_a = -\lambda_a I \) and \( B_a = I \). Then \( y \) grows **unbounded** for all \( \lambda_a \in (0, 1) \).

The overall system is

\[
\begin{bmatrix}
\dot{d} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix}
-\lambda_a I & I \\
L_s & -L_s
\end{bmatrix} \begin{bmatrix}
d \\
y
\end{bmatrix} \Rightarrow \det[s^2 I + (L_s + \lambda_a I)s + (\lambda_a - 1)L_s] = 0
\]

which is unstable for \( \lambda_a < 1 \).

There are numerous choices of \( F_a, B_a \) that leads to instability.

\( \Rightarrow \) need to make the system robust to any possible attacks.
Table of Contents

1 Introduction to Cooperative Control
   - Background
   - Standard Results
   - Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   - Passive and Passivity-Short (PS) Systems
   - Plug & Play Networked Operation of PS Systems
   - Application to Attitude Synchronization
   - Control of Interconnected Heterogeneous Systems Using an Information Graph

3 Resilient Networked Control Against Attacks
   - A Dynamic Attack Model
   - Robustification: Virtual Nodes and Synthetic Anchors
   - Robustification of Cooperative Systems Against Attacks
Example: A Two-Node Robust Cooperative System

Consider a network with two nodes interconnected with a hidden layer:

\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} + \beta
\begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
d_1 \\
d_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} - \beta
\begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
\]

which can be rewritten as

\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{z}_1
\end{bmatrix} =
\begin{bmatrix}
-1 & \beta \\
-\beta & -1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
z_1
\end{bmatrix} +
\begin{bmatrix}
1 & -\beta \\
\beta & 1
\end{bmatrix}
\begin{bmatrix}
y_2 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
d_1 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{y}_2 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
-1 & \beta \\
-\beta & -1
\end{bmatrix}
\begin{bmatrix}
y_2 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
1 & -\beta \\
\beta & 1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
z_1
\end{bmatrix} +
\begin{bmatrix}
d_2 \\
0
\end{bmatrix}
\]
Transfer function for node 1 from attack/disturbance $d_1$ to $y_1$ is given by

$$\frac{Y_1}{d} = \frac{s + 1}{s^2 + 2s + 1 + \beta^2}$$

The resonance frequency is $1 + \beta^2$. Hence, $\beta \uparrow$ yields resonance frequency $\uparrow$.

Robustness design against attacks: 1) superiority of information access; 2) design virtual nodes so they do not impact normal operation of the system; 3) automatic activation when attacks appear anywhere in the system; 4) synthetic anchors to maintain system stability while state estimation is in progress; 5) Mitigation measures embedded
1 Introduction to Cooperative Control
   • Background
   • Standard Results
   • Consensus Laws

2 Nonlinear Plug-and-Play Control Design
   • Passive and Passivity-Short (PS) Systems
   • Plug & Play Networked Operation of PS Systems
   • Application to Attitude Synchronization
   • Control of Interconnected Heterogeneous Systems Using an Information Graph

3 Resilient Networked Control Against Attacks
   • A Dynamic Attack Model
   • Robustification: Virtual Nodes and Synthetic Anchors
   • Robustification of Cooperative Systems Against Attacks
Networked system nodes $y$ ($\Sigma_s$), virtual nodes $z$ ($\Sigma_h$), observation nodes ($\Sigma_o$):

$$
\dot{y} = -L_s y + \beta K z + L_s d,
$$
$$
\dot{z} = -L_h z - \beta G y,
$$
$$
\dot{d} = F_a d + B_a y,
$$

where:

$\beta > 0$: design parameter

**Lemma:** If $d = 0$, $y \rightarrow \mathbf{1} \nu_{s1}^T x(0)/(\nu_{s1}^T \mathbf{1})$ as $t \rightarrow \infty$ provided that:

$$
\Gamma_h G = K^T \Gamma_s, \text{ and } \nu_{s1}^T K = 0,
$$

where $\Gamma_s = \text{diag}\{\nu_{s1}\}$ and $\Gamma_h = \text{diag}\{\nu_{h1}\}$.

**Note:** $G \mathbf{1} = 0 \implies$ consensus value of $z$ is not impacted by $y$

**Sketch of proof:** Lyapunov candidate $V(y, z) = y^T \Gamma_s y + z^T \Gamma_h z$
Robust Design of Cooperative Systems

System:
\[ \dot{y} = -L_s y + \beta L_s z + L_s d, \]
\[ \dot{z} = -L_h z - \beta \Gamma_h^{-1} L_s^T \Gamma_s y, \]
\[ \dot{d} = F_a d + B_a y, \]

**Theorem:** \( y \in \mathcal{L}_\infty \) for all possible choices of \( F_a \) and \( B_a \) and, by increasing \( \beta \), \( y \) converges to an arbitrarily small neighborhood

\[
\lim_{t \to \infty} y(t) = \frac{\nu^T_{s1} y(t_0)}{\nu^T_{s1} 1} 1 + (I + F_a^{-1} B_a + \beta^2 M_h)^{-1} \left[ (c_1 + \beta c_2) 1 - F_a^{-1} B_a \frac{\nu^T_{s1} y(t_0)}{\nu^T_{s1} 1} 1 \right],
\]

where \( \Gamma_h^{-1} L_s^T \Gamma_s = L_h M_h \), and \( c_1 \) and \( c_2 \) are some constants.

**Sketch of proof:** \( V = \beta \tilde{y}^T \Gamma_s \tilde{y} + \beta \tilde{z}^T \Gamma_h \tilde{z} + d^T P_a d + 2 \tilde{z}^T \Gamma_h d, \)
\[ \dot{\tilde{y}} = -L_s (\tilde{y} - \beta \tilde{z} - d), \]
\[ \dot{\tilde{z}} = -L_h (\tilde{z} + \beta M_h \tilde{y}), \]
\[ \dot{d} = F_a d + B_a \tilde{y}. \]
Identification of Stealthy Attacks

When there is no attack

\[ y \rightarrow \mathbf{1} \nu_{s1}^T y(t_0) / (\nu_{s1}^T \mathbf{1}), \quad z \rightarrow \mathbf{1} \nu_{h1}^T z(t_0) / (\nu_{h1}^T \mathbf{1}). \]

Under attacks, states \( y, z \) deviate from above values by \( \tilde{y}^e, \tilde{z}^e \)

Let \( \mathcal{A} \) with \( \dim(\mathcal{A}) = m < n \) be a set of nodes being attacked and

\[ d^e = (\tilde{x}^e - \beta \tilde{z}^e) - c_1 \mathbf{1} \]

which is not unique.

If \( m \leq \lfloor n/2 \rfloor \), the set \( \mathcal{A} \) can be found uniquely by solving \( \hat{c}_1 \) in

\[ \hat{c}_1 = \arg\min_{c_1} \| (\tilde{x}^e - \beta \tilde{z}^e) - c_1 \mathbf{1} \|_{l_0} \]

where \( \| y \|_{l_0} \) is the number of non-zero elements in vector \( y \).

**Note:** \( \nu_{s1}, \nu_{h1} \) can be estimated distributively.
Plug & play control of passivity-short systems:


SO(3) control


Robustification using competitive interaction


Distributed estimation:


Q&A

Thanks!