Dynamics of the Walking Stick Insect

F. Pfeiffer, H.-J. Weidemann, and P. Danowski

In order to gain insight into the biological, constructional conceptions of walking as a basis for future technical developments, the gait dynamics of the walking stick insect (Carausius morosus) is investigated. It is modeled by a rigid multibody system consisting of 19 bodies (1 central body and 6 legs with 3 segments each) and forming multiple closed kinematic chains corresponding to the regarded gait pattern. Thus, the changing structure during the walking motion yields a dynamic system with time-varying topology. The results of the computer simulation are compared with measurements of the insects, confirming the possibility of modeling natural walking motion by mechanical means.

Introduction

Despite intensive research investigations in motion analysis, many principles used by nature still are unknown. With the goal of gaining knowledge of the construction principles in nature and eventually using them for the technical design of walking robots, this article deals with modeling the kinematics and kinetics of the walking stick insect (Carausius morosus).

Regarded in a mechanical analysis, the moving apparatus of the walking stick insect forms an elastic multibody system with various kinematic chains [12], which open and close periodically during the motion cycle. Thus, biological and technical moving systems are dynamic systems with time-varying topology and alternating degrees of freedom. In general, this fact leads to a complex structure of the equations of motion.

In a first step, this article deals with a rigid multibody system including 19 bodies (central body plus 6 legs with 3 segments each) and a single gait pattern on a horizontal plane.

The resulting distance constraints of the closed kinematic chains form a condition for determining the joint coordinates and their time derivatives for the different legs. Determination of the joint torques and contact forces (constraint forces) will be done using an algorithm introduced in the following.

A six-legged "Adaptive Suspension Vehicle" (about the size of a small truck) has been built to illustrate some of the characteristics of legged motion [14]. The construction principles of the moving apparatus in this and smaller machines [5], [10], [11], are based more on technical concepts with cartesian-orientated rotating axis rather than on biological concepts. The resulting gait movements seem rather inflexible with low speed limits. It is thus reasonable to copy design concepts from nature, where especially six-legged insects (hexapods) offer some attractive solutions. Furthermore, these insects have the advantage of a simple, symmetric gait movement: Three legs form a stable tripod while the remaining legs swing into the subsequent tripod position. As a living prototype for a technical walking machine the walking stick insect Carausius morosus (see Fig. 1) has been chosen for this study. It has been intensively investigated by biologists since 1921 [6], [8]. The following characteristic features of the Carausius morosus provide some advantages for biological research and mechanical modeling:

- slow insect movement (max. 2-3 steps/second),
- orderly gait pattern,
- relatively large size (ca. 70 mm) with some distance between leg joints
- legs have three joints instead of four encountered by other insects.

The motion on a horizontal plane is characterized by only two different gait patterns [8]:

- Tripod gait, a very orderly gait pattern in which front and rear leg on the left side together with the middle leg on the right side move in unison and form the left tripod, while the remaining legs conversely form the right tripod. Thus, either three or six legs have ground contact simultaneously.
- Quadrupod gait, a very complicated gait pattern, where all six legs move with different phase characteristics.

In a first step, this article deals with an investigation of the tripod gait on a horizontal plane.

Mechanical Model

Masses and Geometry

The regarded mechanical model for the walking stick insect is a rigid multibody system consisting of the central body and six legs with three segments and joints each. The central body has three degrees of freedom for translation and three for rotation. Furthermore, three joint angles per leg give an additional 18 degrees of freedom (DOF), bringing a total number of 24 DOF to the investigated system. Data specification of masses and geometry are taken from [6]. Table I shows a survey of fundamental data. Fig. 1 shows the mechanical equivalent model (one of six legs is drawn).

Kinematics

Four different coordinate frames are introduced:

- the inertial coordinate frame \( I \), in which the \( \lambda \) axis is parallel to the direction of motion and the \( \zeta \) axis is directed vertically upwards,
- the reference frame \( R \) which is paraxial to \( I \) and shifted with the constant average translational speed \( w \) relative to \( I \),
the body-fixed frame $K$, whose $x$ axis marks the longitudinal axis of symmetry of the central body. It is shifted relative to the reference system by the three translational degrees of freedom, $(q_1, q_2, q_3)$, and rotated by the three Karden angles $(\alpha, \beta, \gamma)$.  

- The 6 Coxa - frames with origins in the connecting joints $C_i$ between the central body and the legs (index $i$ denotes the leg: $l \in \{fl,ml,hr,fr,ml,lr\}$; $f$: front, $m$: middle, $h$: hind, $l$: left, $r$: right). (In biological research, Coxa denotes the first leg segment.) The spatial orientations of these six frames relative to the body-fixed $K$-frame are obtained by a set of elementary rotations:
  - Rotation about the $x$-axis by the time-invariant angle $\phi$.
  - Rotation about the obtained intermediate $y$-axis by the time-invariant angle $\psi$.
  - Rotation about the $z$-axis whose orientation relative to the $K$-frame is time-invariant, by angle $\omega$, denoting the first degree of freedom of each leg.

The three segments of each leg lie in the $x\omega\gamma$-plane, which will be called leg plane in the following. The $x$-axis marks the longitudinal axis of the first segment, $\beta$, and $\gamma$ denote relative angles between the second and first and the third and second segment, respectively (see Fig. 1).

To analyze the kinematics of the regarded gait pattern, we investigate the movements of the tarsi relative to the body of the insect, measured in horizontal and upright projection for discrete times $t, j \in \{1, \ldots, \ell\}$ by [6]. Tarsus denotes the low end of the last leg segment, the tarsi movements relative to the body thus are described by the vectors $r_{cm}$, (see Fig. 1). Phase characteristics of the leg movement have been investigated by [8]; the following computations are based upon a tripod gait on a horizontal plane, the phase lag between the movements of the left tripod and the right tripod being exactly half the gait period denoted by $T$. The motion of the central body can be obtained in following subsequent steps: the vectors $r_{cm}$, locating the contact points $T_i$ in the body-fixed $K$-frame, are obtained by adding the vector from the center of mass to the connecting joint $C_i$ of the leg $l$, $r_{cm}$ (which has time-invariant components in the $K$-frame), to the vector of the relative tarsus location,

$$ r_{cm} = r_{cm} + r_{cm}. $$

A left standing triangle is described by the vectors $r_{cm}, l \in \{fl,ml,hr\}$, a right standing triangle by the vectors $r_{cm}, l \in \{fr,ml,lr\}$ accordingly. During motion on a horizontal plane these triangles are alternately fixed in the inertial $x,y$-plane. Thus, the location and orientation of the $K$-frame relative to a given $l$-frame can be derived inversely. Relative to the $R$-frame the motion of the central body is then described by three generalized coordinates of translation $(q_1, q_2, q_3)$, where

$$ r_{cm}(t) = \{(q_1, q_2, q_3)\}, $$

and three generalized coordinates of rotation $(q_4, q_5, q_6)$. The joint angles $\alpha, \beta, \gamma$ are derived by means of inverse kinematics. The vector of generalized coordinates $q \in \mathbb{R}^6$ is composed of the motion coordinates of the central body $q_1$ to $q_6$ plus the joint angles:

$$ q^T = (q_1, q_2, \ldots, q_6, \alpha, \beta, \gamma). $$

It is reasonable to approximate each of the components of $q$ by a Fourier series, since the motion of the insect appears smooth and harmonic. Using only the first three frequencies of the Fourier

| Table I |
|---|---|---|
| Mass [mg] | Length [mm] | $\theta$ [mm] |
| Main body | 872.4 | 72 | 5 |
| 1st leg segment | 3.3 | 1.7 | 1.5 |
| 2nd leg segment | 9.1 | 12.3 | 1.0 |
| 3rd leg segment | 2.2 | 11.6 | 0.5 |

...
motion are derived from d'Alemberts principle [4]:
\[ \sum_{i} \delta F_i (\delta \dot{r} - \dot{d}F_i) = 0. \] (5)

When regarding a rigid multibody system, the evaluation of (5) simplifies to
\[ \sum_{i} \int_{0}^{\infty} \delta F_i (\delta \dot{r} - \dot{d}F_i) \, dt = 0. \] (6)

where: \( r \) denotes the vector locating the center of mass of the \( p \)th body, \( J_{p} \) the Jacobian matrix of translation, \( F_{p} \) denotes the Jacobian matrix of rotation, \( F_{p} \) active external forces, \( T_{p} \) active external torques. Applying (6) to the walking stick insect yields the equations of motion:
\[ M\ddot{q} = h + J_{p}^T (m\ddot{r} - F_{p}) + J_{p}^T (I_{p} \ddot{\omega} + \ddot{\omega} \omega - T_{p}) \] (7)

where \( M \in \mathbb{R}^{24x24} \) denotes the mass matrix, \( q \) is given according to (3), and \( F_{p} \in \mathbb{R}^{24} \) is the vector of contact forces.

\[ F_{p} = \{ F_{x_1}, F_{y_1}, \ldots, F_{x_n}, F_{y_n}, F_{z_1}, \ldots, F_{z_n} \} \] (8)

where \( h \in \mathbb{R}^{24} \) denotes the generalized influence of gravitation, \( J_{p} \in \mathbb{R}^{24x24} \) is composed of the jacobian matrices of translation of the contact points \( T_{p} \), in columns, \( J_{p} \in \mathbb{R}^{24x24} \) accordingly projects the torques \( T_{p} \) into the space of the generalized coordinates \( q \). The vectors \( F_{p} \) and \( T_{p} \) are unknown, so that there are 36 unknown quantities with 24 equations.

**Optimal Force Distribution**

With all legs in ground contact, 6 closed kinematic loops have to be taken into account, yielding an infinite number of solutions for the contact force distribution. For a simulation of the system in the case of unknown accelerations \( \ddot{q} \), the active torques \( T_{p} \) must be given and Lagrange-multipliers must be used to include the distance constraints of the contact points on the ground in the equations of motion. With a complete analysis of the kinematics, however, the desired gait pattern of the walking stick insect is given over time, \( q(t) \), and resulting accelerations \( \ddot{q} \) are easily acquired by differentiating (4) twice with respect to time. In the case of the walking stick insect, the vectors \( F_{p} \) and \( T_{p} \) remain unknown. Some supplementary assumptions are therefore necessary to determine torques and forces for the based gait pattern \( q(t) \). In [13] the hypothesis of a zero interaction force field is introduced.

The components of each pair of contact forces projected on the joining line of their corresponding contact points are equal. The structure of this force field (equilibrating force field) is analogous to an helicoidal vector field and is the solution of the minimum norm problem for the planar components of the contact forces. This assumption provides the desired additional equations for a complete solution, but yields high active torques in the outer leg joints for a stick insect even in quasistatic computations. If one considers human straddle-legged standing, practical experience shows that the legs are pressure-loaded only, stretching the ground between the feet by evoking interaction forces [9] add an interaction force field with zero net resultant to the equilibrating force field to obtain further conditions and to optimize the solution. Another solution of the distribution problem is specified when regarding a quadratic criterion \( C \) in dependance of the unknown forces (8) and torques (9), which, e.g., may describe the elastic deformation energy of the movement apparatus, and demanding

\[ C := \int (T_{p}, F_{p}) \rightarrow \min. \] (10)

This criterion does not presume a special arrangement of the contact forces and takes the bending load of the legs into account, yielding low active torques and low bending loads especially in the outer, thin leg segments. With the bending moments \( M_{y} \) and \( M_{z} \), Young's modulus \( E \) and the geometrical moments of inertia \( I_{y} \) and \( I_{z} \), the deformation energy \( V_{s} \) caused by bending loads for thin beams of length \( L \) is given by

\[ V_{s} = \frac{1}{2E} \int_{0}^{L} \frac{M_{y}(x)}{I_{y}} \frac{M_{z}(x)}{I_{z}} dx \] (11)

Neglecting the small leg masses, the bending moments \( M_{y}, M_{z} \) depend on the active torques \( T \) and the passive foot forces \( F \). Thus the deformation energy \( V_{s} \) itself is a function of \( q, T, F \).
\[ C = V_v = f(q, T, F) \]
\[ = \left( \sum_{i=1}^{n} c_i T_i \right) + \left( \sum_{m=1}^{n} d_m F_m \right). \]  

The coefficients \( c_i \) and \( d_m \) have to be computed for each time step using equation (11). Furthermore, the equations of motion have to be satisfied by the active forces and torques. The Lagrange Multiplier Theorem uses Lagrange multipliers \( h \) to couple these secondary factors to the optimal criterion \( C \). In the following a short algorithm is introduced to determine the active torques and forces required for the chosen gait pattern of the walking stick insect. With \( n \) denoting the dimension of the vector of generalized coordinates \( q \) and \( p \) denoting the dimension of the vectors of forces and torques, the \( n \) rows of the equations of motion (7) are coupled to the criterion \( C \) via Lagrange multipliers \( \lambda \), thus forming the Lagrange function \( L \). Here, \( j_r, j_m, m \) and \( h \) denote the components of the corresponding matrices in (7):

\[
L := C + \sum_{i=1}^{n} \lambda_i \left( j_r \cdot T_i \right) + \sum_{i=1}^{n} \left( j_m \cdot \dot{q} - h_i \right) + \sum_{k=1}^{k} \lambda_k \sum_{i=1}^{n} \left( u_{ik} \cdot F_i \right) \tag{13}
\]

With three legs in ground contact instead of six, the contact forces on the lifted feet vanish to zero quantities,

\[ U \cdot F = 0, \quad U \in \mathbb{R}^{3 \times n}. \]

This distribution matrix \( U \) consists of zeros and elements corresponding to the \( k \) inactive contact force components \( \lambda \). These secondary restrictions are added by the Lagrange multipliers \( \lambda_{in} \) to \( \lambda_{out} \). The dimension of the vector \( \lambda \) thereby increases from \( m \) to \( n + k \). According to the Lagrange Multiplier Theorem the value of the criterion \( C \) is optimal for vanishing partial derivatives of \( L \) with respect to all unknown quantities \( \lambda, T, F \):

\[
\frac{\partial L}{\partial \lambda_i} = 0 \land \frac{\partial L}{\partial T_j} = 0 \land \frac{\partial L}{\partial F_k} = 0 \tag{15}
\]

where \( i \in \{1, \ldots, p\} \) and \( j \in \{1, \ldots, n + k\} \).

Written in matrix notation, (15) leads to a system of linear equations of the dimension \( 2p + n + k \), where the matrices \( C, D \in \mathbb{R}^{(p \times n)} \) are derived by partial differentiation of the criterion \( C \) with respect to the unknown forces and torques,

\[
C := \frac{\partial C}{\partial T}; \quad D := \frac{\partial C}{\partial F}. \tag{16}
\]

The vector of Lagrange multipliers \( \lambda \) is separated into \( \lambda_i, \lambda_j, \lambda_k \) as follows:

\[
\lambda_i := [\lambda_{i1}, \ldots, \lambda_{in}]^T, \tag{17}
\]

and

\[
\lambda_j := [\lambda_{j1}, \ldots, \lambda_{jn}]^T, \tag{18}
\]

with the contact forces. A qualitative accordance in some instances different gait patterns, limiting any comparisons to qualitative character. Even small deviations in insect behaviour will produce discrepancies in the time history of the contact forces. A qualitative accordance in direction and magnitude especially of the thrusting action of the hind legs can be observed, while the front legs are subjected to low forces only. This result may be explained by the tactile task of the front legs during normal motion and is also a consequence of the geometric position as is shown by the simulation.

**Experiments**

Experiments were conducted by Prof. Cruse regarding the leg movements of adult stick insects[6]. A small piece of the surface of a walking path was cut out and then fixed to a force meter, which consists of strain gauges and corresponding amplifier and signal processing electronics. When the walking insect touches the platform in this location the contact force is measured. The force meter was orientated in three Cartesian directions and a set of experiments was performed for each direction. Thus the force components are recorded separately.

**Contact Forces**

Fig. 5 shows the contact forces of the front, middle, and hind leg in the walking plane (\( x \)- and \( y \)-components). The based criterion \( C \) aims to minimize the bending load in the outer leg segments. Unfortunately, the measurements probably include different gait velocities and in some instances different gait patterns, limiting any comparisons to qualitative character. Even small deviations in insect behaviour will produce discrepancies in the time history of the contact forces. A qualitative accordance in direction and magnitude especially of the thrusting action of the hind legs can be observed, while the front legs are subjected to low forces only. This result may be explained by the tactile task of the front legs during normal motion and is also a consequence of the geometric position as is shown by the simulation.

**Discussion**

The mathematical model of an insect has been developed for studies on control strategies in nature. The dynamics of the legs were not neglected; simulation studies of high...
speed insect locomotion show that the joint torques due to leg mass inertia increase significantly. For determining contact forces the criteria of minimum power in leg joints and minimum overall energy consumption were tested next to the demand for minimum bending load. In the special case of walking insects with long, thin leg segments the criterion demanding minimum bending load leads to a qualitative accordance of the computer simulation results with the measurements [6]. Still further work done by biologists is to be examined in order to strengthen the thesis of this control law and to compare results produced by simple neural network controllers [1],[2] with this explicit control law.

References


Friedrich Pfeiffer obtained the Dipl.-Ing. degree in 1961 from the Technische Hochschule Darmstadt, where in 1965 he received the Dr.-Ing. degree with a thesis in the aerodynamics field. He was research engineer in the technical staff of the Bölkow GmbH (1969-1975), Director of the Mechanics Division of MBB (1975-1976), Technical Assistant of Dr. L. Bölkow (1975-1976), and Managing Director of the Bayerchemie and Director of the Apparatus Development Section (1980-1982). Since 1983 he has been Ordinarius of the Mechanics Institute of the Technical University Munich. His teaching and research interests center around dynamics and control of rigid and elastic mechanical systems in general, and elastic robots and rotating machinery in particular.

Hans-Jürgen Weidemann obtained his Dipl.-Ing. degree in 1988 at the Institute of Mechanics of the Technical University Munich. Since then he has been Scientific Assistant of this Institute. His research interests include the dynamics and active control of walking machines.

Peter Danowski (deceased) received his Dipl.-Ing. degree with a thesis on Dynamics of Walking Stick Insects in 1989 at the Institute of Mechanics of the Technical University Munich. He was Scientific Assistant of this Institute when he was killed in a traffic accident in 1990.